Evolutionary Graph Theory and Structural Power

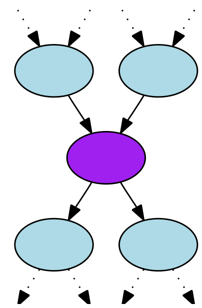
Lee Worden
http://two.ucdavis.edu/~worden
worden@berkeley.edu

Networks in Political Science,
June 13–14, 2008, Cambridge, Mass.

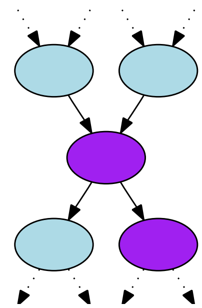
Evolutionary Graph Theory (EGT)

E. Lieberman, C. Hauert, M. Nowak, "Evolutionary dynamics on graphs," *Nature* 433:312–316 (2005)

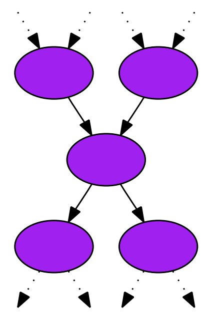
A directed graph (network) represents a population of individuals. All are identical (resident fitness $= 1$) except for one fitter variant (variant fitness $= r > 1$).



Individuals reproduce at a rate proportional to fitness, replacing a randomly selected neighbor in the network each time.



Eventually the variant type either vanishes (extinction) or replaces the old type (fixation).



The probability of fixation depends on network structure.

One model has multiple interpretations

Biology interpretation

How evolutionary dynamics are affected by population structure:

> "organisms" occupy sites;
> offspring propagate to downstream sites;
> mutant type reaches fixation in the population with some probability

Social interpretation

"We can ask, for example, which networks are well suited to ensure the spread of favorable concepts" (Lieberman et al., 2005):

> concepts are held by individuals;
> concepts propagate along social connections to downstream individuals;
> the group reaches consensus with some probability

The published claim:
centralization improves collective intelligence

“A selection amplifier, like a star structure or a scale-free network, will enhance the spread of favourable ideas arising from any one individual. Notably, scientific collaboration graphs tend to be scale-free” (Lieberman et al., 2005).

Star or scale-free networks have special “hub” individuals who are much more central than most individuals (they have more connections, and more pathways go through them).

These hubs appear to aggregate the inputs they receive from many neighbors and amplify the distinction between the two types, making the network more likely to make the “right” choice.

Centralization has a down side as well

Hubs are weak points: Interference with a “hub” individual can debilitate or even disconnect a network, making these networks lack robustness to disruption (as in for instance Barabási, *Linked: The New Science of Networks* (2002)).

But also,

Hubs concentrate power: As we will see, these individuals’ preferences have a disproportionate influence on the outcome of this “deliberation” process. Efficiency is traded off against democracy, and if these individuals are corruptible or otherwise unreliable, the network is more vulnerable than a more distributed one would be.

Research questions

In the social interpretation of this model, r is how strongly each individual favors the new concept.

What if r is different for different people? If someone especially favors the idea, their r will be large, and if they are against it, their r will be small.

Does the fixation probability depends more strongly on some nodes' r than others? This would mean that some people's opinions carry more weight than others, because of their position in the network.

If so, which ones? Does influence correlate with centrality in the network? That is, does centralization correspond to a concentration of power?

If so, "enhancing the spread of favorable ideas" would come with a tradeoff — they would be the ideas favorable to a select few, and what if those few aren't representative?

“Influence” in EGT models

Probability of fixation ρ can be found (by analysis or simulation) for a given graph structure and set of fitnesses.

In this context “influence” of an individual is a measure of how sensitive ρ is to variation in that individual’s r value.

Measure sensitivity of fixation probability to r_i:

$$\mathrm{Sens}(i) = \frac{\partial \rho}{\partial r_i}.$$

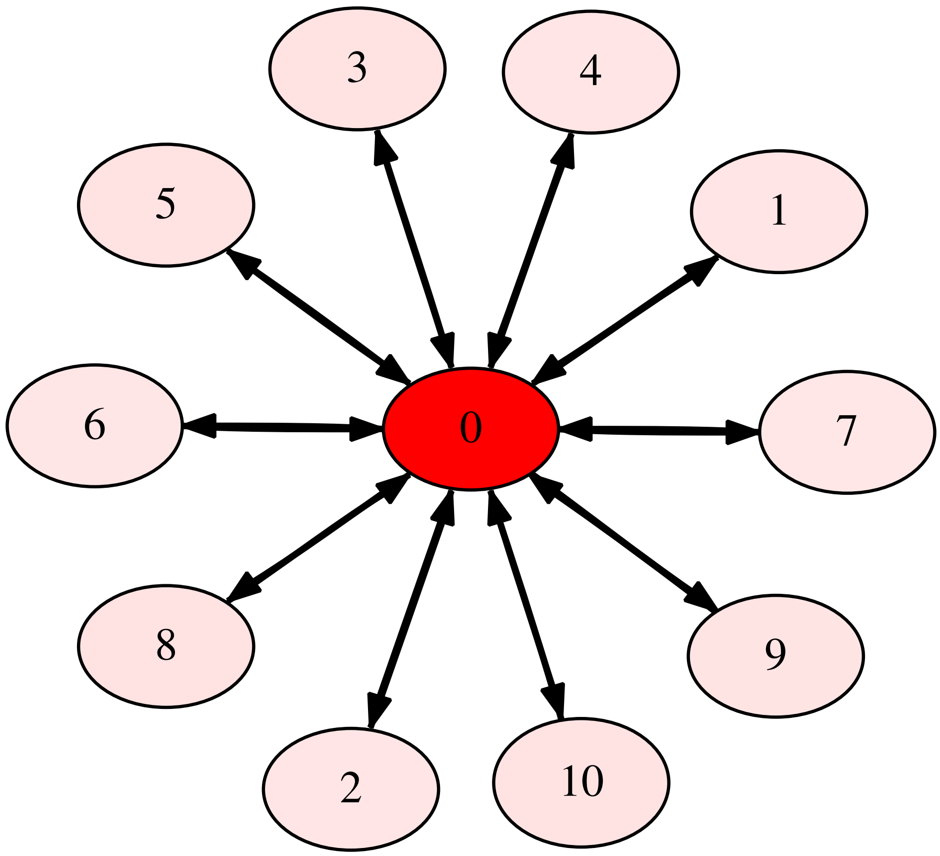
Define influence as how that sensitivity compares to the average:

$$\mathrm{Inf}(i) = \frac{\mathrm{Sens}(i)}{\frac{1}{n}\sum_j \mathrm{Sens}(j)}$$

(n = number of individuals).

If everyone were equal (as in the completely connected case), the $\mathrm{Inf}(i)$ would all be 1.

Influence of nodes in the star network



Center node is much more influential (red) than peripheral nodes (see next slide).

Analysis of star network

On a star with m peripheral nodes, let

$$r_0 = \text{variant fitness at the center node}$$

and

$$\begin{aligned} r_p &= \frac{1}{m}\sum_{i\neq 0} r_i \\ &= \text{average variant fitness on the periphery.} \end{aligned}$$

r_0 and r_p are about the same size. Probability of fixation is

$$\rho = \frac{1-(r_0 r_p)^{-1}}{1-(r_0 r_p)^{-m}}$$

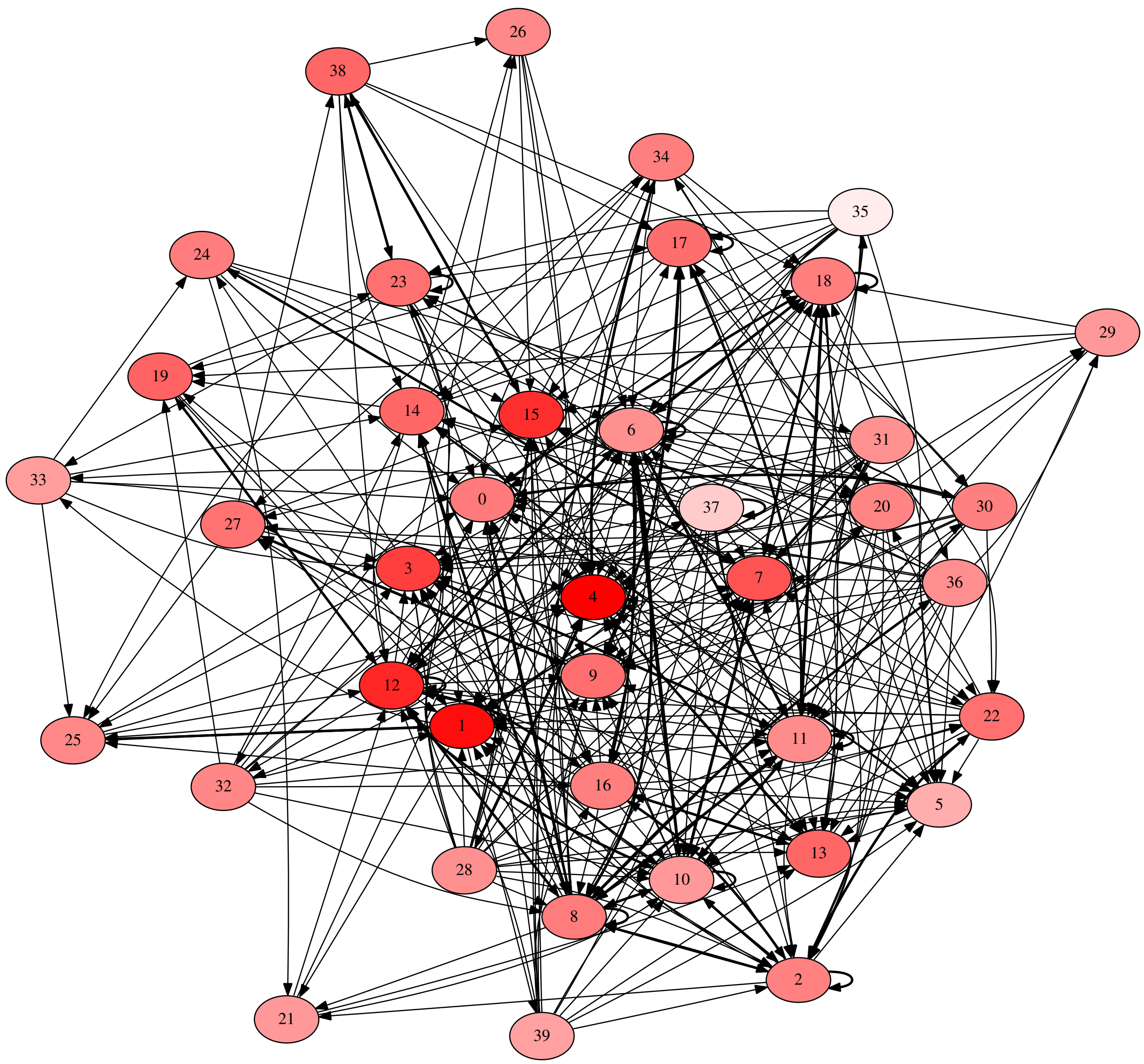
as m grows large.

As a consequence, variation in r_0 has much more effect on the fixation probability than any other r_i:

$$\begin{aligned} \frac{\partial\rho}{\partial r_0} &= r_p \frac{\partial\rho}{\partial(r_0 r_p)} \\ i \neq 0: \quad \frac{\partial\rho}{\partial r_i} &= \frac{1}{m} r_0 \frac{\partial\rho}{\partial(r_0 r_p)}. \end{aligned}$$

r_0 counts as much as all the other r values put together.

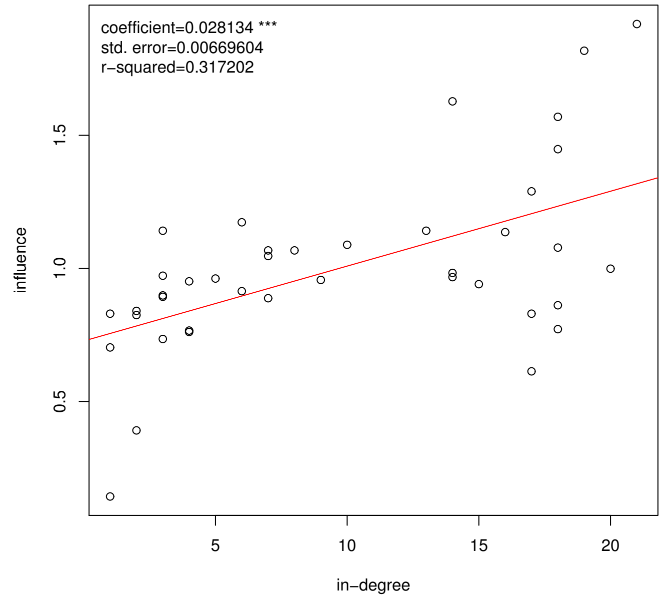
Influence vs. degree and centrality



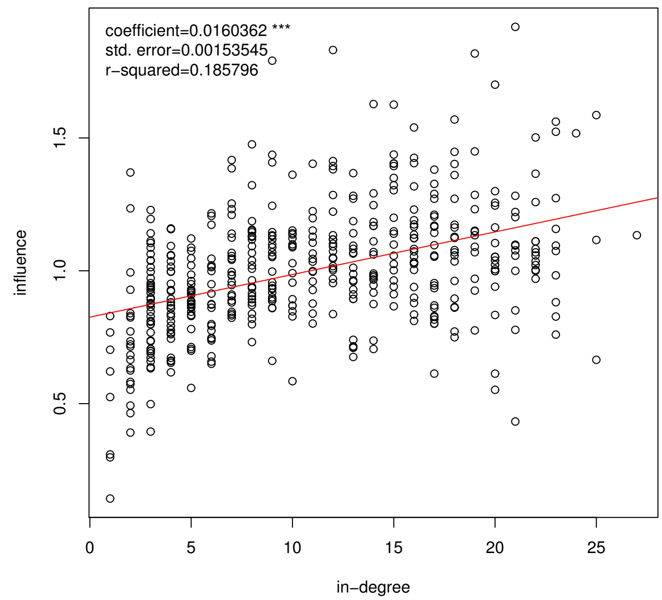
Results following are taken from a sample of 12 power-law-out-degree graphs of 40 nodes.

In degree is the number of arrows entering a node.

Example graph

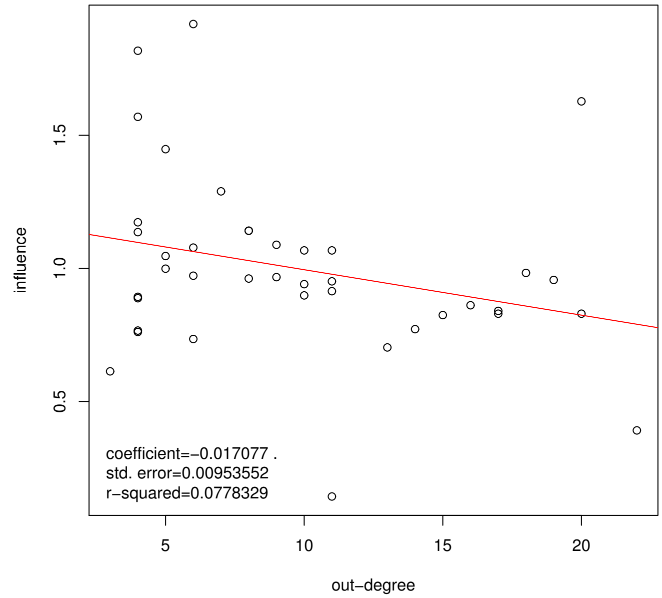


Ensemble of graphs

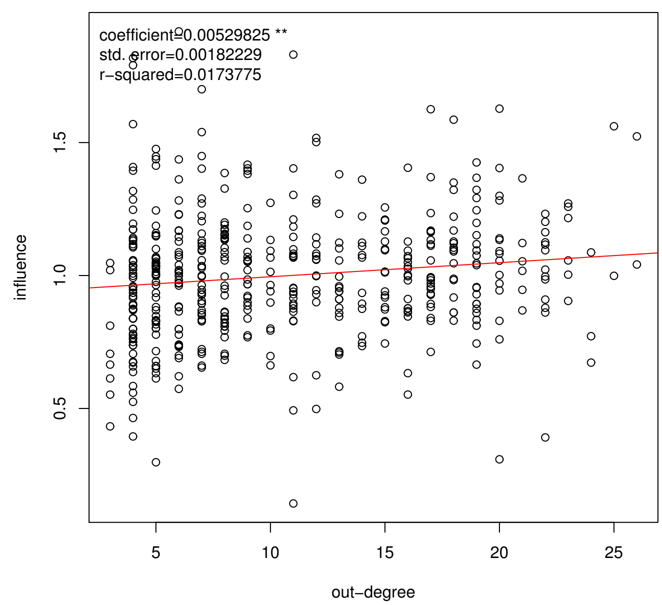


Out degree is the number of arrows leaving a node.

Example graph

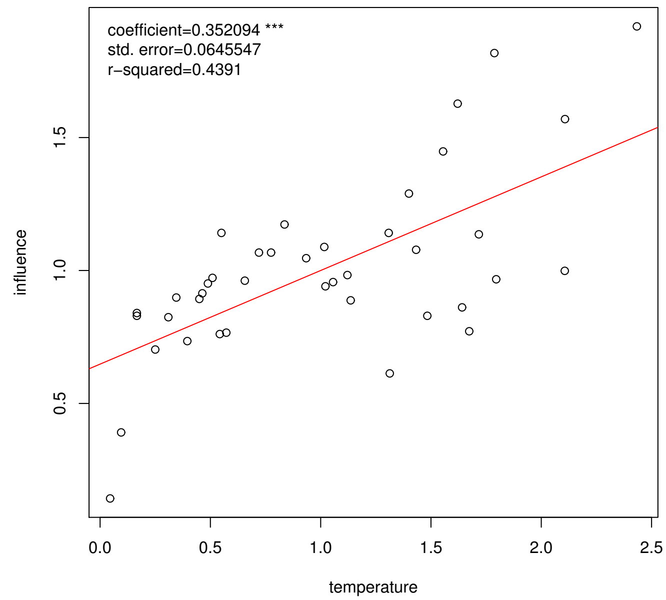


Ensemble of graphs

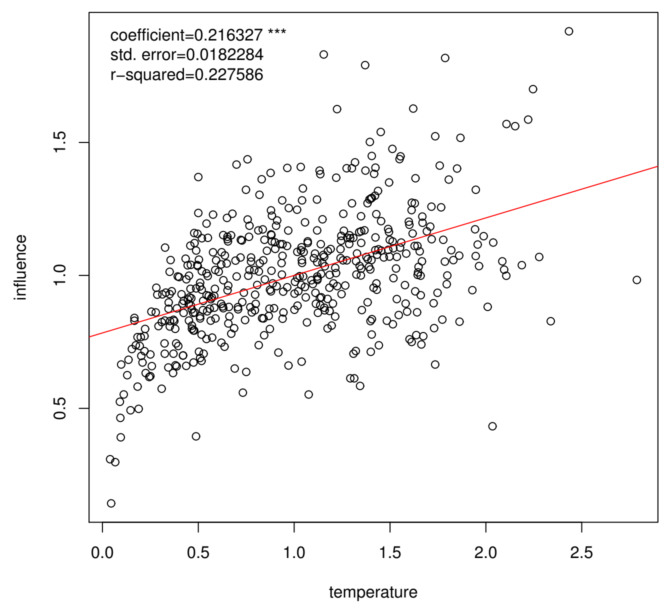


Temperature is like in degree, but weighted so it measures the actual rate of births arriving at a node.

Example graph

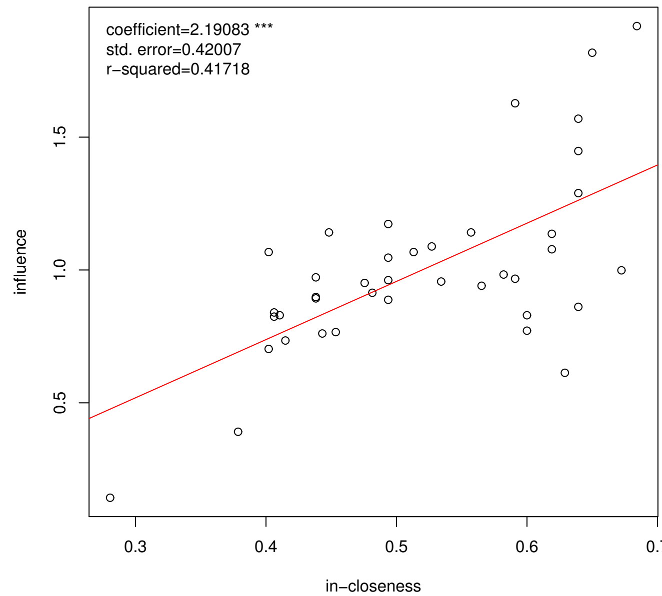


Ensemble of graphs

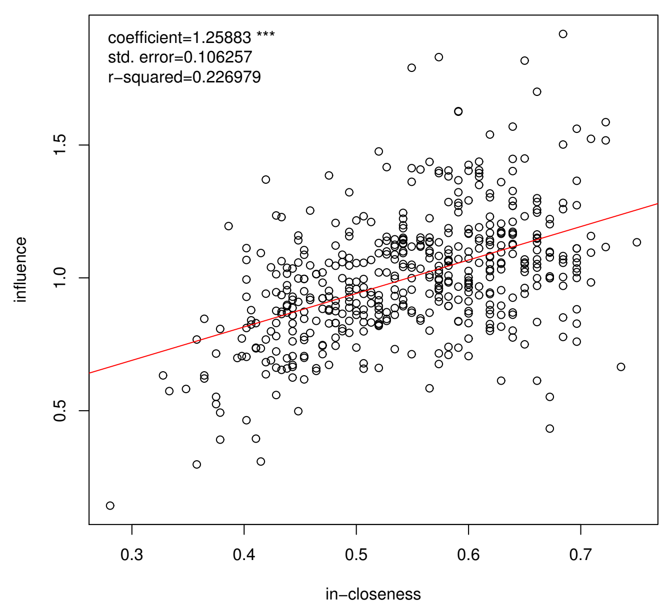


In closeness measures shortness of paths to a node from all the others.

Example graph

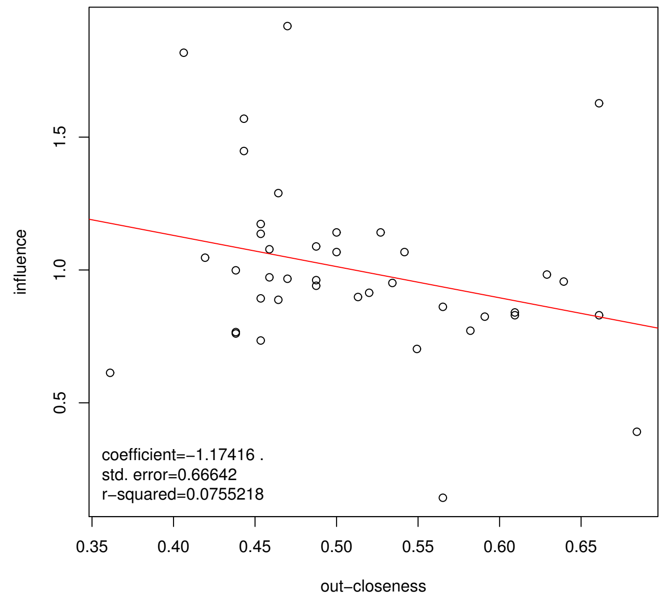


Ensemble of graphs

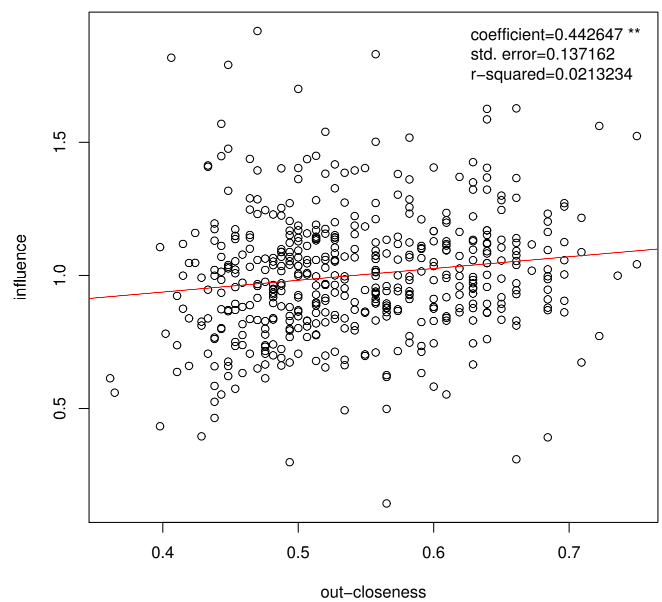


Out closeness measures shortness of paths from a node to all the others.

Example graph

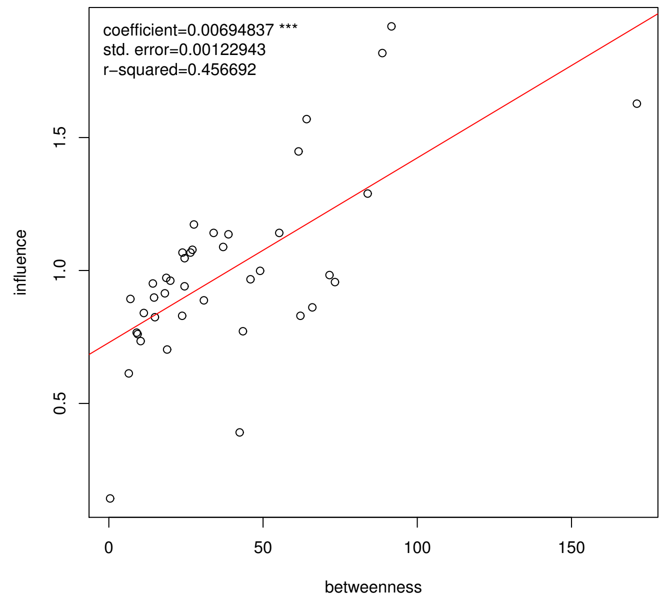


Ensemble of graphs

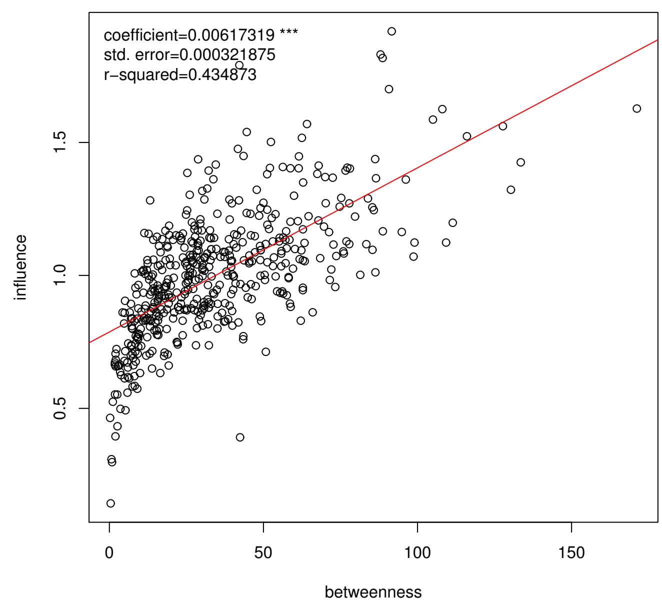


Betweenness is the number of pairwise shortest paths that pass through a node.

Example graph

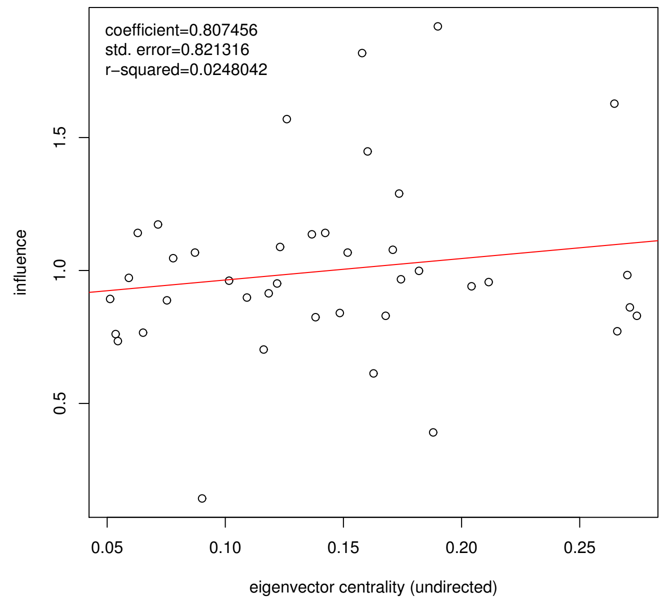


Ensemble of graphs

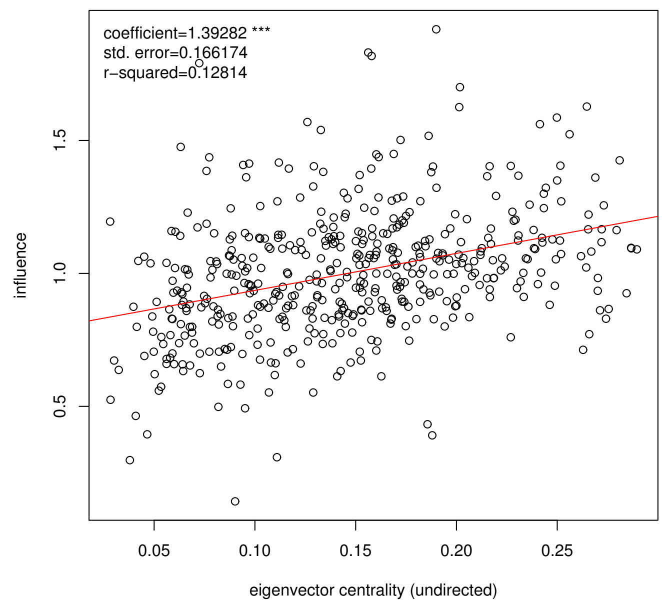


Eigenvector centrality is how much time a random walk would spend at each node (if the graph were undirected).

Example graph

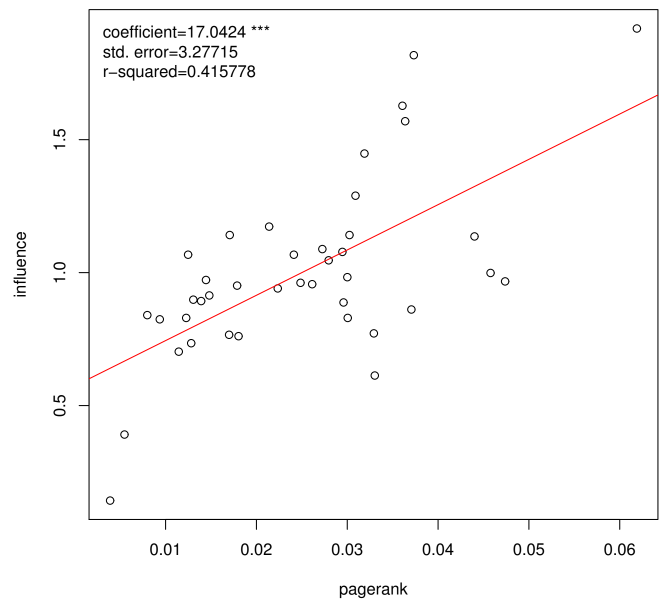


Ensemble of graphs

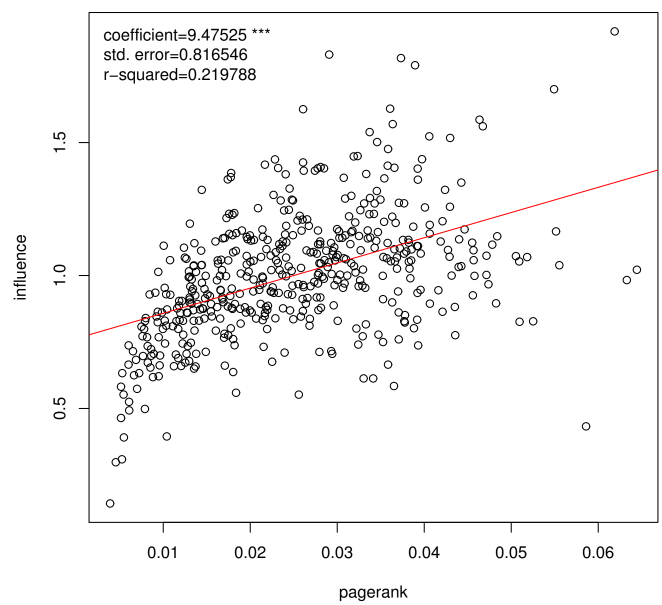


“Pagerank” is a form of directed eigenvector centrality.

Example graph



Ensemble of graphs



Conclusions

In these graphs, influence in the EGT model correlates strongly with a number of centrality measures.

The connections entering a node predict influence much better than the connections leaving it. Influence especially seems to favor nodes that aggregate input from many others.

Many centrality measures predict influence well. This suggests that an individual who is a go-between for communication has a strong influence on the collective deliberation process in this model. These results generalize the result for the star network.

It follows that while centralization increases a network's ability to discern options when all individuals are equally wise and trustworthy, it also concentrates power in a few individuals and discounts others' perceptions. Thus centralization is a risky design choice when individuals' judgement or integrity may falter. This is in addition to the risk that the network may be fragmented by loss of a central individual.

Acknowledgements

My research group and brainstorm partners:
Skye Bender-deMoll, Nathan Murthy, Todd Parsons, Peter Ralph, Eddie Yuen

Special thanks to Joshua Plotkin (U. of Pennsylvania) and his lab for hosting me in fall 2007, and to Iain Boal and Cynthia Peters.

Software: Boost, GNU C++, Igraph, R, Graphviz, LaTeX, Ubuntu Linux